SEDIMENT-SIGNAL INTERACTION

KEY:
Yellow are properties of the Side Scan sonar
Green are adjustments made within the Side Scan Sonar Software
Orange are calculated values/ parameters
Grey are main equations with unsolved variables (ie porosity)
Blue are water properties/properties from the field
Purple are values that we are trying to get to and may need to be iterated
Red are values that can be found in the lab
White are notes, assumptions, and values to be determined.

PROPERTIES

Frequency [hz]

Gain (amplitude)

Check initial gain, panel gain and percentage gain

Source level, SL [dB re 1 μPa @ 1 m]

$$s_0 = 10^{\frac{SL}{20}}$$

Height [m]

Grazing angle (incident grazing angle), θi

Range [m]

Density of water (pore fluid mass density), ρ_w ,ρ_f [g/m^3]

Temperature of water

Speed of sound in water, c_w [m/s^2]

Attenuation, α [dB/m]

Area, A [m^2] calculated from the smallest area you can get data

Directivity function = cos (θ), where theta is the angle from the maximum response axis (MRA). The beam is assumed to be located at the MRA, so this is equal to one. r is the distance from the SSS to the point of measurement.

$$|P_i| = \frac{\sqrt{2}s_0\,|b_x(\theta,\phi)|}{r}$$

Magnitude of the incident pressure $|P_i|$

$$U_i = \frac{A|P_i|^2}{2\rho_w c_w} sin(\theta_i)$$

Incident energy for an area, Ui

SIGNAL-WATER INTERACTION

Scattering due to roughness

Scattering due to sediment volume

Bottom Loss

Interpolation to go between Kirchoff and small-pertubation approximation

Acoustic wavenumber in water, k = 2π/λ [m^{-1}] (or alt. formula ω/c_w)

Angular frequency, ω = 2π*f* [hz]

$$\epsilon = a\sqrt{\frac{\omega\rho_f}{\eta}}$$

The dynamic viscosity of the pore fluid, η [N s/m^2]

$$a = \sqrt{\frac{8\alpha\kappa}{\beta}}$$

Coefficient of permeability, κ [m/s]

Tortuosity, α [dimensionless]

$$T(\in) = \frac{(-\sqrt{i})\,J_1(\epsilon\sqrt{i})}{J_0(\epsilon\sqrt{i})}$$

Bessel Function values (from table)

$$F(\epsilon) = \frac{\frac{\epsilon}{4}T(\epsilon)}{1-\frac{2i}{\epsilon}T(\epsilon)}$$

$$\alpha = 1-r(1-1/\beta)$$

shape factor, r, r= 1/2 for spherical particles, otherwise 0<r<1 [dimensionless]

$$P(\theta) = \sqrt{\kappa^2 - (cos\theta)^2}$$

$$\kappa = \frac{1+i\delta}{v}$$

Defined as the "complex wave number in the sediment divided by the real wave number in water"

ASSUME: sediment type parameter (BACKCHECK)

Speed of sound in sediment, c [m/s]

(CHECK) Sound speed ratio, *v* = c / c_w

(CHECK) Loss parameter, δ = IM(k_s) / k_s

Sediment mass density, ρ, Use Peff [g/m^3]

Density of water (pore fluid mass density), ρ_w ,ρ_f [g/m^3]

NOTE: this value connects to most everything because of the effective density and will be carried through the calculations as a variable and interated

Fractional Porosity (sediment porosity), β

$$\rho_{eff}(\omega) = \rho_f\left(\frac{\alpha(1-\beta)\rho_s + \beta(\alpha-1)\rho_f + \frac{i\beta\rho F\eta}{\rho_f\omega\kappa}}{\beta(1-\beta)\rho_s + (\alpha-2\beta+\beta^2)\rho_f + \frac{i\beta F\eta}{\omega\kappa}}\right)$$

Total mass density, ρ = βρ$_f$+(1- β)ρ$_s$ [g/m^3]

Sediment particle mass density, ρ$_s$ [g/m^3]

SMALL-ROUGHNESS PERTUBATION APPROXIMATION

density ratio = **ρ /ρ$_w$**

$$G = \left(\frac{1}{\rho}-1\right)\left[cos(\theta_i)\,cos(\theta_s)\,cos(\phi_s) - \frac{P(\theta_i)\,P(\theta_s)}{\rho}\right] + 1 - \frac{\kappa^2}{\rho}$$

$$W(K) = \frac{w_2}{(h_0K)^{\gamma_2}}$$

$$\sigma_{pr}(\theta_s,\phi_s,\theta_i) = \frac{1}{4}k_w^4|1+R(\theta_i)|^2|1+R(\theta_s)|^2|G|^2W(2k_w\Delta_t)$$

NOTE: for calculating backscatter

$$2k\Delta_t = 2k_w cos(\theta_i)$$

KIRCHOFF APPROXIMATION

Strength of spectrum, w_2 [length^4] (determined in field)

Exponent of the bottom relief spectrum, γ_2 (determined in field, 2 ≤ γ_2 ≤ 4)

Reference length parameter, h_0 = 1 [length]

Bessel Function values (from table)

$$C_h^2 = \frac{2\pi w_2\Gamma(2-\alpha)\,2^{-2\alpha}}{h_0^{\gamma_2}\alpha(1-\alpha)\Gamma(1+\alpha)}$$

$$q = 2k_w^2\Delta_z^2C_h^2(2k\Delta_t)^{-2\alpha}$$

$$\alpha = \frac{\gamma_2}{2} - 1$$

$$\sigma_{kr}(\theta_s,\phi_s,\theta_i) = \frac{|R(\theta_{is})|^2}{8\pi}\left(\frac{\Delta^2}{\Delta_z\Delta_t}\right)^2 * \int_0^\infty e^{-qu^{2\alpha}}J_0(u)\,udu$$

Gamma Function, Γ()

$$\sigma_{br}(\theta_s,\phi_s,\theta_i) = \left[\sigma_{kr}^\eta(\theta_s,\phi_s,\theta_i) + \sigma_{pr}^\eta(\theta_s,\phi_s,\theta_i)\right]^{1/\eta}$$

Interpolation

NOTE: This is the monostatic cross section (backscatter) if incident angle, θ_i = scattered angle, θ_s; and azimuth, ϕ_s = π.

$$S_b(\theta_s,\phi_s,\theta_i) = 10log[\sigma_{br}(\theta_s,\phi_s,\theta_i) + \sigma_{br'}(\theta_s,\phi_s,\theta_i)]$$

Strength of bistatic crossection [dB]

CHECK: strength of bistatic crossection = Ui - BL

Bottom loss, BL [db] = -20log(|R(θ)|)

Complex plane-wave reflection coefficient, R(θ_i)

ITERATE

Ratio of the complex amplitudes of the reflected and incident waves; R(θ_i) = S_b / U_i

ASSUME: incident angle, θ_i = scattered angle, θ_s; and azimuth, ϕ_s = π. This is the monostatic case (backscatter).

$$\Delta = \sqrt{\Delta_t^2 + \Delta_z^2}$$

$$\Delta_z = \frac{1}{2}(sin(\theta_i) + sin(\theta_s))$$

$$\Delta_t = \frac{1}{2}\left[(cos(\theta_i))^2 - 2cos(\theta_i)\,cos(\theta_s)\,cos(\phi_s) + (cos(\theta_s))^2\right]^{1/2}$$

$$R(\theta_i) = \frac{Z_s - Z_w}{Z_s + Z_w}$$

$$Z_w = \frac{\rho_w c_w}{cos(90-\theta_i)}$$

$$Z_s = \frac{\rho c_s}{cos(\theta_s)}$$

$$\theta_s = sin^{-1}\left[\frac{c_s sin(90-\theta_i)}{c_w}\right]$$

Try calculated density here and also effective density

CHECK: Index of Impedance, IOI = ρ* (c_s/c_w): use to check porosity and density estimations

SCATTERING DUE TO SEDIMENT VOLUME

$$\Delta k = k_s\left|4\Delta_t^2 + \{Re[P(\theta_i) + P(\theta_s)]\}^2\right|^{1/2}$$

Inhomogeneity exponent, γ_3 = 4(?)

fluctuation ratio, μ = compressibility/ density fluctuations in the sediment (det. in field)

$$W_{\rho\rho}(k) = \frac{w_3}{(h_0k)^{\gamma_3}}$$

Inhomogeneity Strength, w_3 = strength of sediment inhomogeneity spectrum [cm^3]at wave number = 1 cm^{-1}

$$\sigma_v = \frac{\pi}{2}k_s^4\left|\mu\kappa^2 + cos(\theta_i)\,cos(\theta_s)\,cos(\phi_s) - P(\theta_i)\,P(\theta_s)\right|^2W_{\rho\rho}(\Delta k)$$

$$\sigma_{br'}(\theta_s,\phi_s,\theta_i) = \frac{|1+R(\theta_i)|^2|1+R(\theta_s)|^2\sigma_v}{2k_w\rho^2Im[P(\theta_i)+P(\theta_s)]}$$

ASSUME: initial value (BACK CHECK)

Reference length parameter, h_0 = 1

$$P(\theta) = \sqrt{\kappa^2 - (cos\theta)^2}$$

$$\kappa = \frac{1+i\delta}{v}$$

Wave number in sediment, k_s [m^{-1}]

Loss parameter, δ = IM(k_s) / k_s

Sound speed ratio, *v* = c_s / c_w

Speed of sound in sediment, c_s [m/s]